# 57SIG – Issue 492. Spatiotemporal formalization about the presence of parts

In the 57th CIDOC CRM & 50th FRBR/LRMoo SIG Meeting, WS presented HW. Details [below](#_HW_Document_by).

**Discussion points**:

The document describes what it means to have a part addition for the spatiotemporal relations of the thing that gets added to another thing.

Describing these relations can get pretty messy, whence stems the need to change the formalism used to represent FOL axioms (especially the ones with the more intricate semantics), to use functions and named FOL clusters to do so. Alternatively, rendering the relations in prose seems to work quite well.

**Decisions**:

* The change of the formalism is a different issue. It can be discussed among MD, CEO, and WS as a separate issue.
* **HW**: CEO and MD to revise the document, summarize it for the broader audiences. Maybe color code it as well.

## HW Document by WS:

This is what I attempt to do here. For example, I try to describe what a property such as “P111 added” means for the physical extents of the thing that is added and the thing it is added to.

**P8, P12, P110, P111, P112, P113**

A little refresher for the properties mentioned in issue 492:

P12, P110, P111, P112, P113 in the property hierarchy:

E18 Physical Thing P31i was modified by E11 Modification

E18 Physical Thing **P110i was augmented by** E79 Part Addition

E18 Physical Thing **P112i was diminished by** E80 Part Removal

E5 Event **P12 occurred in the presence of** E77 Persistent Item

E7 Activity P16 used specific object E70 Thing

E79 Part Addition **P111 added**E18 Physical Thing

E80 Part Removal **P113 removed** E18 Physical Thing

Part Removal is more difficult than Part Addition, so I leave it for a continuation of this homework.

P8 and P7 are mentioned in issue 492, but neither P8 not P7 are discussed here. For the sake of completeness:

E4 Period **P8 took place on or within** E18 Physical Thing

is a shortcut of

E4 Period **P7 took place at** E53 Place P156i is occupied by E18 Physical Thing

**Named FOL clusters**

Because the axioms in this homework tend to be very long, I am defining some *named FOL clusters*. For example:

u is presence of x during t  ⇔  E18(x) ∧ E52(t) ∧ E93(u) ∧ P195i (x,u) ∧ P164(u,t)

(Physical Thing *P195i had presence* Presence *P164 is temporally specified by* Time-Span)

y is physically part of x during t

⇔ E18(x) ∧ E18(y) ∧ E52(t) ∧ the presence of y is within the presence of x during t

⇔ E18(x) ∧ E18(y) ∧ E52(t) ∧ (∃u,w) [u is presence of x at t ∧ w is presence of y at t ∧ P10(w,u)]

⇔ E18(x) ∧ E18(y) ∧ E52(t) ∧ (∃u,w) [E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ P10(w,u)]

Note:

* As can be seen in this example, duplicate class definitions can be removed.

**P46 is composed of**

E18 Physical Thing P46 is composed of (forms part of) E18 Physical Thing

The existing axiom

The existing axiom:

P46(x,y) ⇒ (∃u,z,w) [E93(u) ∧ P195i (x,u) ∧ E52(z) ∧ P164(u,z) ∧ E93(w) ∧ P195i (w,y) ∧ P164(w,z) ∧ P10(w,u)]

If one moves (∃u,w) into the (∃t) bracket to minimise the scopes of the quantifiers, it can be written as

P46(x,y) ⇒ (∃t) [E52(t) ∧ (∃u,w) [E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ P10(w,u)]]

With named FOL clusters, it is equivalent to

P46(x,y) ⇒ (∃t) [y is physically part of x during t]

**Avoiding edge cases that can make the axiom meaningless**

FOL axioms in CIDOC CRM tend to be self-contained. For example, there is no general statement about the conditions under which the spatial projection of a period exists in a given reference space. However, a concrete statement such as “x P7 took place at y” implies not only that the spatial projection z (in the same reference space as y) is within y, but also that this z does indeed exist.

Likewise, there are no general statements about the existence or non-existence of empty time-spans or empty presences. The right-hand side of the P46 axiom if trivially true if empty time-spans and empty presences are allowed. Thus, we need to rule out that the time-span is empty or outside the existence of y. (This implies that it is also within the existence of x, and that the presences of x and y are non-empty.)

P46(x,y) ⇒ (∃t) [t has positive duration ∧ t is within the existence of y ∧ y is physically part of x during t]

Additional named FOL clusters:

t has positive duration ⇔ E52(t) ∧ (∃r) [E52(r) ∧ P86(r,t) ∧ ¬(r=t)]

t is within the existence of y ⇔ E52(t) ∧ E18(y) ∧ (∃s) [s is the duration of the existence of y ∧ P86(t,s)]

s is the duration of the existence of y ⇔ E52(s) ∧ E18(y) ∧ (∃v) [E92(v) ∧ P196(y,v) ∧ P160(v,s)]

The resulting self-contained axiom:

P46(x,y) ⇒ (∃t) [E52(t) ∧ (∃r) [E52(r) ∧ P86(r,t) ∧ ¬(r=t)] ∧ (∃s) [E52(s) ∧ (∃v) [E92(v) ∧ P196(y,v) ∧ P160(v,s)] ∧ P86(t,s)] ∧ (∃u,w) [E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ P10(w,u)]]

Since we already know that s and v exist (and are unique) for any physical thing y, we can move them to the left-hand side, i.e. turning the (∃s) on the right-hand side into an implicit (∀s) on the left-hand side:

P46(x,y) ∧ E92(v) ∧ P196(y,v) ∧ E52(s) ∧ P160(v,s) ⇒ (∃t) [E52(t) ∧ (∃r) [E52(r) ∧ P86(r,t) ∧ ¬(r=t)] ∧ P86(t,s) ∧ (∃u,w) [E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ P10(w,u)]]

(i.e. if P46(x,y) and v is the Spacetime volume of y with duration s ⇒ … )

* Alternatively, we can require the presence of y to be non-empty. Then the presence of x is also non-empty, and t must have positive duration and be within the existence of y. This could be written like this:

P46(x,y) ⇒ (∃t) [E52(t) ∧ (∃w,u) [E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ (∃r) [E52(r) ∧ P86(r,t) ∧ ¬(r=t)] ∧ E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ P10(w,u)]]

However, I find the version with the conditions for t instead of w conceptually more clear.

**Using symbols**

The sheer length of the self-contained axiom would make it desirable to move at least some part of the heavy lifting elsewhere. For example, one could introduce a symbol “∅” in P86 with

E52(∅) ∧ (∀r) [E52(r) ⇒ P86(∅,r)]

and

t has positive duration ⇔ ¬(t=∅)

P46(x,y) ∧ E92(v) ∧ P196(y,v) ∧ E52(s) ∧ P160(v,s) ⇒ (∃t) [E52(t) ∧ ¬(t=∅) ∧ P86(t,s) ∧ (∃u,w) [E93(u) ∧ P195i (x,u) ∧ P164(u,t) ∧ E93(w) ∧ P195i (y,w) ∧ P164(w,t) ∧ P10(w,u)]]

(Or state that there is no time-span with this property.)

→ Martin during the presentation: let’s generally state that time-spans have non-zero length

Was it in E52 or in P86? Conceptually it seems to belong in E52, but in addition to the statement in a scope note we need an axiom like

E52(t) ⇒ (∃r) [E52(r) ∧ P86(r,t) ∧ ¬(r=t)]

which would be the first non-trivial axiom in the FOL of any class.

**Using functions**

We could also get rid of the (∃u,w). With a minimal, uncontroversial existence axiom for presences, namely

E92(x) ∧ E52(s) ∧ P160(x,s) ∧ E52(t) ∧ ¬(t=∅) ∧ P86(t,s) ⇒ (∃u) [E93(u) ∧ P195i (x,u) ∧ P164(u,t)]

(i.e. for a Spacetime Volume x with duration s and a non-empty time-span t within s, the presence of x for the time-span t exists; in fact, it is also non-empty)

Since presences are always unique, we can define a function

Presence u = F195(Physical Thing x, Time-Span t)

whose domain includes at least all (x,t) where t is non-empty and within s, which is sufficient for the P46 axiom. Thus we can write it as:

P46(x,y) ∧ E92(v) ∧ P196(y,v) ∧ E52(s) ∧ P160(v,s) ⇒ (∃t) [E52(t) ∧ ¬(t=∅) ∧ P86(t,s) ∧ P10(F195(y,t), F195(x,t)) ]

With an additional function for the duration of the Spacetime Volume of a Physical Thing, which also exists and is unique:

duration s = F160(Physical Thing y)

P46(x,y) ⇒ (∃t) [E52(t) ∧ ¬(t=∅) ∧ P86(t, F160(y)) ∧ P10(F195(y,t), F195(x,t)) ]

This way we can avoid introducing variables such as v and s that we are not really interested in.

**P46 is a shortcut of a E79 Part Addition**

P46 is a shortcut with the long path via E79 Part Addition:

P46(x,y) ⇐ (∃z) [part addition z has added y to x]

part addition z has added y to x ⇔ E79(z) ∧ E18(x) ∧ E18(y) ∧ P110i(x,z) ∧ P111(z,y)

P46(x,y) ⇐ (∃z) [E79(z) ∧ P110i(x,z) ∧ P111(z,y)]

Note:

* (∃z) is generally not needed in non-strong shortcuts, so it could also be written as P46(x,y) ⇐ E79(z) ∧ P110i(x,z) ∧ P111(z,y)
* P46 is probably \*not\* a shortcut of E80 Part Removal since the conditions for the existence of y are different: There doesn’t have to be a time when y exists and forms part of x.

**Analysis of E79 Part Addition**

“part addition z has added y to x” implies:

at start of z:

* x and y already exist (y is not created in the Part Addition)
* y is not part of x (in the sense of P46)

at end of z:

* x and y still exist
* y is part of x (in the sense of P46)

Taken together:

* z is temporally (properly) within the “existence” condition states of x and y
* y has a non-empty “part of x” condition state p:
  + p is within the existence of x and y
  + p includes or starts with the end of z (i.e. starts after the start but before or with the end of z, ends after the end of z)
  + y is physically part of x during p

For now, I leave out the “not part of x” condition state.

**The self-contained axiom**

part addition z has added y to x

⇒  (∃c) [c is “existence” condition state of x ∧ z is properly within c]

∧ (∃d) [d is “existence” condition state of y ∧ z is properly within d]

∧ (∃p) [y has non-empty (“part of x”) condition state p

∧ p starts after the start of z ∧ p starts before or with the end of z ∧ p ends after the end of z

∧ y is physically part of x during p]

c is “existence” condition state of x

⇔ x defines STV with duration t ∧ E3(c) ∧ P44(x,c) ∧ E55(existence) ∧ P2(c, existence) ∧ P4(c,t)

x defines STV with duration s

⇔ E92(vx) ∧ P196(x, vx) ∧ E52(s) ∧ P160(vx, s) ⇔ s = F160(x)

y has (“part of x”) condition state p

⇔ E18(y) ∧ E3(p) ∧ P44(y,p) ∧ p has positive duration ∧ p is within c

(P44 has condition, P2 has type, P4 has time-span)

In all, the axiom looks somehow like this:

P46(x,y) ∧ E79(z) ∧ P110i(x,z) ∧ P111(z,y)

∧ E92(vx) ∧ P196(x, vx) ∧ E52(s) ∧ P160(vx, s)

∧ E92(vy) ∧ P196(y, vy) ∧ E52(t) ∧ P160(vy, t)

⇒

(∃c,d,p,u) [E3(c) ∧ P44(x,c) ∧ P4(c,s) ∧ P176i(z,c) ∧ P185(z,c)

∧ E3(d) ∧ P44(y,d) ∧ P4(d,t) ∧ P176i(z,d) ∧ P185(z,d)

∧ E3(p) ∧ E52(u) ∧ P160(p,u) ∧ ¬(u=∅) ∧ P44(y,p) ∧ P175i(p,d) ∧ P184(p,d)

∧ P176i(p,z) ∧ P173(p,z) ∧ P185i(p,z)

∧ P10(F195(y,u), F195(x,u)) ]

Note:

* “z is within the existence of x and y” could be expressed via their time-spans, but even then it is difficult to catch the fact that z is is properly within the existence of x and y. (I use “properly within” in the sense of “starts after the start AND ends before the end”, which is not the mathematical meaning, which is closer to “starts after the start OR ends before the end”. The mathematical meaning could still be expressed with time-spans.)
* Account for the possibility that e.g. a button can be part of a jacket several times.
* I haven’t found a way of comparing the temporal relationships of p and z without temporal primitives and only via their time-spans. Thus I needed to find a way from a physical thing to something where temporal primitives are allowed. E2 Temporal Entity would have worked, but E3 Condition State seemed more fitting since it is a state after an event. However, I am not sure, and this might be abusing the respective classes.

To be continued.